Study the relation between band gap value and lattice constant of MgTi$_2$O$_4$

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**Abstract**

As an accurate technique, spectroscopic ellipsometry (SE) is utilized to study the dielectric function ($\varepsilon = \varepsilon_r + i\varepsilon_i$) of spinel oxide MgTi$_2$O$_4$ (MTO) thin films with good orientation. C-axis lattice length and Ti ions valence state are characterized by X-ray diffraction and X-ray photoelectron spectroscopy, respectively. Here, we find that c-axis lattice constants have the positive correlation with band gaps which are obtained by fitting the spectra of $d^2(\varepsilon^2_{\varepsilon_i})/dE^2$ via standard critical points model in MTO, and this phenomenon can be revealed by the Ti ions valence state. Moreover, the band gap value of MTO measured by experiment is consistent with the result of first-principles calculations.

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**1. Introduction**

Spinel oxides with AB$_2$O$_4$ structure are ranked as one of the richest groups. With different physical properties including spin fluctuations [1–3], charge ordering [4,5] and orbital ordering [6], spinel oxides with AB$_2$O$_4$ structure generate intriguing functionalities such as superconductivity [7], ferroelectricity [8,9], magnetostriiction [9,10] and so on. Excepting these interesting physical properties, spinel oxides also exhibit wide applications in electronics, catalysis and electrochemical science such as batteries [11–13], fuel cells [14–16] and electrolyzers [17]. As a typical correlated spinel oxide, MgTi$_2$O$_4$ (MTO) has been extensively investigated due to the special electric structure. In this compound, Ti ions are surrounded by six oxygens forming a TiO$_6$ octahedron. Ti-3d levels split to lower triply degenerate levels t$_{2g}$ and higher doubly degenerate levels e$_g$ under local cubic environment. In MTO, Ti$^{3+}$ indicates a free electron in t$_{2g}$ orbital, whereas a metal-insulator transition (MIT) along with cubic to tetragonal structure transition, happens at about 260 K, which has attracted many research groups to use transport measurements to explore its physical mechanism [18,19]. However, optical reports are far less than transport measurements. Actually, optical properties play an important role in studying novel materials and potential applications.

As one nondestructive optical method, spectroscopic ellipsometry (SE) technique measures the amplitude and phase information simultaneously. Thus, SE has become a suitable way to determinate the optical constants N = n + ik or complex dielectric function $\varepsilon = \varepsilon_r + i\varepsilon_i$ [20,21], which don’t need to convert data from other optical experimental data, such as reflectivity measurement. Generally, $\varepsilon_r$ represents the loss of materials and through fitting the second derivative spectra of $\varepsilon_i (d^2(\varepsilon^2_{\varepsilon_i})/dE^2)$ can quantitively research the electronic structure via standard critical point (SCP) model [22]. Jiang et al. [23] have researched the optical transition of CH$_3$NH$_3$PbI$_3$ perovskite by the second derivative via SE.

As known to us, physical properties of materials are decided by lattice crystalline. As another hotspot, studying the relationship between lattice constants and physical properties of materials has widely attracted many researcher’s attentions [24–26]. Especially, the length of c-axis lattice constant always affects the electronic structure. Very recently, Feng et al. [27] have given the positive correlation between T$_{c0}$ (zero resistance superconducting critical temperature) and c-axis lattice constant in FeSe thin films. It should be noted that previous researches of MTO are mostly...
polycrystalline powder, which cannot study deeply its physical features. However, we have fabricated the MTO thin films with single orientation on (001)-oriented MgAl2O4 (MAO) substrates by pulsed laser deposition (PLD) technique, and thin films are differed by the pulsed laser energy during growth process. X-ray diffraction (XRD, D8ADVANCE), X-ray photoelectron spectroscopy (XPS, ThermoFisher SCIENTIFIC) and atomic force microscopy (AFM, Nanosurf) are employed to characterize their crystal structures, Ti ions valence state and surface morphology, respectively. C-axis lattice constants are calculated by Bragg’s formula, and SE (GESS, SOPRA) is utilized to get the optical constants. The interband transition energy value is analyzed by SCP model. In addition, we use the first-principles calculations to confirm the band gap value for MTO. What’s more, in the viewpoint of Ti ion valence state, the relationship between c-axis length and transition energy is uncovered.

2. Experimental and theoretical methods

2.1. Fabrication

In PLD technique, the wavelength of laser for fabricating MTO thin films is 248 nm that decides the single photon energy. However, the laser energy per pulse is determined by the quantity of photons. Therefore, in our work, the only variable is pulsed laser energy during synthesizing MTO samples. The detailed synthesis conditions are listed by Table 1.

2.2. Characterization

Lattice structure of MTO thin films are measured by XRD with X-ray wavelength λ = 1.54 Å owing to the fact that X-ray source is Cu Kα1. Commercial Jade software is utilized to calculate the c-axis lattice constants. XPS is used to determine the Ti ions valence state. Crystal grain surface topographies are characterized by AFM. SE with a rotating analyzer explores samples’ optical properties at room temperature. To get smoother ellipsometric data, the spectral resolution is set to 5 nm in the 300–800 nm wavelength range at an incident angle of 75.07.

2.3. Theoretical calculations

Based on the density functional theory (DFT), we combine the first-principles calculations to research density of states (DOS) and band structure of MTO compound. The calculation results come from Cambridge Sequential Total Energy Package (CASTEP) code. Gradient approximation (GGA) is used to deal with the exchange-correlation potential. According to the coverage test, a 400 eV energy cutoff is adopted for the plane wave expansion of the electronic wave function, and a 2 × 4 × 4 Monkhorst-Pack grid is employed over the Brillouin zone [28–30].

3. Results

3.1. Lattice structure and surface morphology

Fig. 1 displays the XRD 2θ spectra of MTO thin films grown on (001)-oriented MAO substrates. The measuring range is 25–65°. (004) diffraction peaks of MTO and MAO are clearly arrowed in this pattern, which indicates single orientation and good epitaxy for all samples. The (004) diffraction peaks of MTO around 42.5° are enlarged in the inset. It can be distinctly observed that the difference of pulsed laser energy during growth makes the position of (004) diffraction peak vary, which demonstrates that the lattice constants change. This phenomenon of lattice constant as a function of pulsed laser energy is related by cation stoichiometry deviations [31].

Lattice constant of c-axis can be calculated by the Bragg’s formula, and the values are listed in Table 2. From this Table, 8.4639 Å of MTO_1 is the longest one. Other study has pointed out that lattice constant of MgTi2O4 compound is 8.503 Å [32] where the Ti ions valence state is absolutely +3. Moreover, another work has reported that Ti ions valence state decreases as c-axis lattice constant increases [33]. Therefore, we consider the Ti ions valence state deviates +3 among MTO_1, 2 and 3, and meanwhile, MTO_1 possesses the lowest valence state of Ti ions as it owns the longest c-axis in our work.

Because Ti ions valence state is sensitive to c-axis lattice constant and changes with stoichiometry [34]. Thus, XPS technique is utilized to analyze the valence state of Ti ions, and Fig. 2 displays the binding energy of Ti 2p level. The green lines represent the peak maximum of Ti 2p3/2, which are 457.83, 458.26 and 458.59 eV for MTO_1, 2 and 3, respectively. These values are all lower than 458.70 eV of TiO2 and higher than that of 457.40 eV in Ti2O3, indicating that the valence state of Ti ions is between +3 and +4 [35]. According to the binding energy, MTO_1 owns the lowest Ti ions valence state and MTO_3 is the highest one.

AFM is a powerful tool for characterizing thin film surface

Table 1

<table>
<thead>
<tr>
<th>Sample name</th>
<th>Growth temperature (°C)</th>
<th>Pulsed laser energy (mJ)</th>
<th>Sputtering frequency (Hz)</th>
<th>Deposition time (min)</th>
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<td>MTO_1</td>
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Table 2

<table>
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<th>Sample name</th>
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<td>MTO_1</td>
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<td>MTO_2</td>
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<tr>
<td>MTO_3</td>
<td>8.4396</td>
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morphology. Fig. 3 displays the scanning images on $1 \times 1 \mu m^2$ area. Homogeneous grains tightly arrange in the testing area, and smooth surface is clearly observed for MTO_1, 2 and 3, respectively. High quality samples open a credible door for systematic investigation of their optical properties.

3.2. Data of spectroscopic ellipsometry

Fig. 4 exhibits the simulated (solid lines) and experimental (hollow dots) data for ellipsometric angle amplitude and phase of the polarized reflected light from the surface of MTO thin films. According to AFM results, smooth surfaces are obtained. Thus, a three-layer model is built to express the optical structure (ambient/ MTO/MAO substrate). Drude and three Lorentz oscillators are used to describe MTO-layer, and the same optical structure and dispersion law are utilized for these different lattice length samples. The Drude and Lorentz oscillator dispersion model are given by Table 3.

Where, $P$, $D$ and $E$ are the polarization, inverse of the plasma wavelength and mean free path respectively for Drude law. $A$, $L_0$ and $\gamma$ are the intensity, central wavelength and width of absorption peak respectively for Lorentz oscillation. The best fitting parameters of Drude and Lorentz model are listed in Tables 4 and 5.

The film thickness is obtained to be 137.1 nm, 129.7 nm and 158.8 nm for MTO_1, 2 and 3, respectively. In the last, a consistent result between simulated and experimental data is achieved for all samples, see Fig. 4.
4. Discussion

4.1. Optical properties and electronic transition energy of MTO

As seen in Fig. 5, it shows the dependence of refractive index \( n \) and extinction coefficient \( k \) on the photon energy of MTO. Refractive index \( n \) (see Fig. 5 (a)) increases with the incident photon energy. The inset is the Drude component which is extracted from dispersion law. Obviously, the real part of dielectric function \( (\varepsilon_r) \) of Drude model as Y-axis decreases towards the low photon energy in the inset, which indicates that these samples are typical metallic behavior at room temperature [36]. For extinction coefficient \( k \) (Fig. 5 (b)), small shoulders which are generally concerned with electronic transition energy as well as interband critical point energy \( (E_{cp}) \) appear around 2.8 eV for all samples. In addition, there exists an upturn above 3.75 eV. We suspect that there are other transition peaks exceed the testing range.

4.2. Theoretical calculation of electronic transition energy for MTO

Fig. 6 (a) and (b) are the band structure and the (DOS) of MTO, respectively. DOS is composed by the total density of states (TDOS) and the partial density of states (PDOS). As shown in Fig. 6 (b), Fermi level goes through the conduction bands, indicating that MTO is a metal material which is consistent with optical measurement. Based on the band theory, the outermost atomic levels (Mg: 3s2, Ti: 3d4s2 and O: 2s22p4) interact with each other. O-2p levels capture electrons to form lower energy valence band or higher energy conduction band, and Ti-3d levels form lower energy conduction band. Thus, the predominant contribution to TDOS derives from O-2p and Ti-3d levels, demonstrating the strong hybridization between O-2p and Ti-3d levels within the Ti-O octahedra. In addition, \( t_{2g} \) and \( e_{g} \) levels split by crystal field are obvious in Fig. 6 (b). Through the accurate calculations, the band gap between O-2p and Ti-3d levels is 2.6 eV.

4.3. In the view of Ti ions valence state to analyze electronic transition energies and understanding the relationship between c-axis lattice constants and band gaps of MTO

Fig. 7 (a), (b) and (c) exhibit the fitting results of \( d^2(E^2\varepsilon_i)/dE^2 \) dependence on photon energy that are performed by SCP analysis which can obtain accurately the quantitative values of band gap value, \( E_{cp} \). The expression of SCP model is given by

**Table 3**
Drude and Lorentz oscillator dispersion model.

<table>
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<tr>
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<th>Lorentz</th>
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<td>( \varepsilon_i(\omega) = A \omega^2 / (\omega^2 + \omega_L^2) )</td>
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<tr>
<td>( \varepsilon_i(\omega) = A \omega^2 / (\omega^2 + \omega_L^2) )</td>
<td>( \varepsilon_i(\omega) = A \omega^2 / (\omega^2 + \omega_L^2 + \omega_L^2) )</td>
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**Table 4**
The best fitting parameters for Drude dispersion model.

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<th>( E )</th>
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**Table 5**
The best fitting parameters for Lorentz oscillator dispersion model.

<table>
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<th>Peak 2</th>
<th>Peak 3</th>
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<td>( L_0 )</td>
<td>( \gamma )</td>
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<tr>
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**Fig. 5.** (a) Refractive index \( n \) and (b) extinction coefficient \( k \) as a function of photon energy of MTO thin films. The inset in (a) shows the \( \varepsilon_i \) of Drude components which derived from the dispersion law.

**Fig. 6.** (a) Band structure and (b) the density of state of MTO compound.
\[
\frac{d^2 (E^2 \varepsilon)}{dE^2} = \begin{cases} 
    n(n-1)Ae^{i\phi}(E - E_{cp} + i\Gamma)^{n-2}, & n \neq 0 \\
    A e^{i\phi}(E - E_{cp} + i\Gamma)^{-2}, & n = 0
\end{cases}
\]

where \( A, \phi, E \) and \( \Gamma \) represent the amplitude, excitonic phase angle, threshold energy and broadening parameter, respectively. Exponent \( n \) equals to \( 1/2, 0, -1/2 \) and \(-1 \) for three-, two-, one-dimensionality and discrete excitons line shape. In our work, \( n \) is given to \(-1 \). The \( E_{cp} \) are 2.87 eV, 2.85 eV and 2.80 eV for MTO_1, 2 and 3, respectively. 

**Based on the results of XPS, we determine that MTO_1 possesses the smallest valence state of Ti. In this case, compared with other two samples, holes (positive charge) introduced into the system are the least one. Therefore, the hybridization between O-2p and Ti-3d level is the weakest as the Ti-3d level offer the least holes needed to be neutralized by O-2p electron. Thus, MTO_1 has the widest band gap which is associated with experiment result 2.87 eV. In this way, c-axis length should have the positive correlation with band gap.** 

**More surprisingly, the three samples we synthesis do follow this rule, and this conclusion are plotted in Fig. 7 (d). Moreover, we correlate material band gap with its lattice constant, which broaden SE potential application in terms of characterizing crystal structure.**

**4.4. Comparison the transition energy between MTO and Li₂TiO₄ (LTO) compound**

The last but not least, we compare the electronic transition features of MTO and Li₂TiO₄ (LTO) compound. The reason to compare is that, on the one hand, the same spinel crystal structure and similar electronic structure are considered. On the other hand, inspired by the superconductivity with \( T_c = 11 \) K of Ti mixed-valent of LTO [24], MTO is also predicted to own 50 K superconductivity. Thus, they may possess the similar physical properties, such as orbital order.

Based on the previous theoretical calculation of electronic structure about LTO [29], the band gap between O-2p and Ti-3d level is 2.26 eV which is smaller than MTO. This phenomenon can also be explained by Ti ions valence state. +3.5 valence state is considered in LTO in theoretical calculation work, and holes of introducing to LTO system are more than MTO_1. Because MTO_1 is the closest one to +3 where the binding energy of 457.83 eV is very close to 457.4 eV in Ti₂O₃. Therefore, more holes are needed to neutralize by O-2p electrons in LTO than MTO_1. In this case, hybridization between O-2p and Ti-3d level in LTO would be stronger than MTO_1. Therefore, the band gap of LTO is less than MTO_1. It offers a clear reference for choosing an appropriate photoelectric device or potential battery materials [28,37,38] in the future.

**5. Conclusions**

In summary, we present a detailed study on lattice constants, Ti ions valence state, and dielectric functions of high quality MTO thin films. Due to the fact that different laser energy during deposition make the lattice constant change, we extract the positive correlation between lattice constants calculated by Bragg’s formula and band gap values obtained by fitting the SCP model of \( d^2 (E^2 \varepsilon) / dE^2 \). This phenomenon can be well revealed by Ti ions valence state. Our results suggest that the valence state Ti ions plays a critical role on understanding the mechanism of the band gap value from O-2p to Ti-3d level in MTO system, which may further shed light on the potential properties of MTO, such as superconductivity. Meanwhile,
regulating band gap in MTO compound will be an underlying application in photoelectric detector.

Acknowledgments

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References


